ALGEBRA

1. Given that $a=\log_5 35$ and $b=\log_9 35$, show that $\log_5 21=\frac{1}{2b}(2ab-2b+a)$.

2. Find the value of $\log_a\sqrt{bc}$, given that $\log_a bc^3=8$ and $\log_a b^3c^2=10$.

3. Find the common difference, the $n^{th}-$ term and the sum to n terms of the A.P given by $In\,3+In(3^2)+In(3^3)+\ldots$.

4. Solve the inequality: $|x-2|>3|2x+1|$.

5. Solve the inequality: $\frac{x+1}{2x-3}\le\frac{1}{x-3}$.

6. Express $1-5x-2x^2$ in the form $a-b(x+c)^2$, hence, deduce the maximum value of the expression.

7. Find the square root of: $23-4\sqrt{15}$.

8. The geometric mean of two numbers a and b for $(b>a)$ is equal to Four Fifth the arithmetic mean of the two numbers. If $a=6$, find the value of b.

9. If $(x+1)^2$ is a factor of $p(x)=2x^4+7x^3+6x^2+ax+b$. Find the values of a and b, hence solve $p(x)=0$.

10. A polynomial $P(x)$ is a multiple of $(x-3)$ and the remainder when divided by $(x+3)$ is 12. Find the remainder when $P(x)$ is divided by (x^2-9).

11. Find three numbers in an A.P such that their sum is 27 and their product is 504.

12. Solve the simultaneous equations: $\frac{1}{u+v}+\frac{2}{u-v}=8$, $u^2-v^2=\frac{1}{6}$

13. Expand $\frac{1}{\sqrt{1+x}}$ up to the term in x^2 and by letting $x=\frac{1}{4}$, show that $\sqrt{5}\approx\frac{256}{115}$.

14. Find the value of n for which the coefficient of x, x^2 and x^3 in the binomial expansion of $(1+x)^n$ are in an Arithmetic progression.

15. Find the coefficient of x in the expansion of $\left(x+\frac{2}{x^2}\right)^{10}$.

16. Prove by Mathematical induction that: $\sum_{r=2}^{n}\frac{1}{r^2-1}=\frac{3}{4}-\frac{2n+1}{2n(n+1)}$

17. Given that $z=\cos\theta+i\sin\theta$, show that $\frac{z-1}{z+1}=i\tan\frac{\theta}{2}$.

18. Determine n if in the expansion of $(2+3x)^n$ in ascending powers of x, the coefficient of x^{12} is four times that of x^{11}.

19. Describe the locus of the complex number z when it moves in the argand diagram such that $\arg\left(\frac{z-3}{z-2i}\right)=\frac{\pi}{4}$.

20. The complex number $u=2i$. The complex number with modulus 1 and argument $\frac{2}{3}\pi$ is denoted w. Find in the form $x+iy$, the complex numbers w, uw and $\frac{u}{w}$.

ANALYSIS

21. Given that $y=\frac{e^x-e^{-x}}{e^x+e^{-x}}$, show that $\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0$.

22. Evaluate: $\int_0^2\sqrt{(4-x^2)}dx$

23. Express $\frac{6-9x}{27x^3+8}$ in partial fractions and hence find $\int\frac{6-9x}{27x^3+8}dx$

24. Solve the d.e $x\frac{dy}{dx}=1-y^2$, given that $x=2$, $y=0$, find y in terms of x.

25. Solve the differential equation: $x^2\frac{dy}{dx}-xy=3y^2$ given $y=1$ when $x=1$.

26. The volume of a water reservoir is generated by rotating the curve $y=kx^2$ about the y-axis. Show that when the central depth of water in the reservoir is h metres, the surface area A is proportional to h and the volume, v is proportional to h^2.

If the rate of loss of water from the reservoir due to evaporation is λA m^2 per day, obtain a differential equation for h after t days. Hence, deduce that the depth of water decreases at a constant rate.

Given that $\lambda = \frac{1}{2}$, determine how long it will take for the depth of water to decrease from 20 m to 2 m.

27. Given that $y = In\left(x + \sqrt{x^2 + a^2}\right)$, where a is a constant, prove that $\frac{dy}{dx} = \frac{1}{\sqrt{x^2 + a^2}}$ and hence evaluate $\int_0^4 \frac{dx}{\sqrt{x^2 + 9}}$.

28. Find: $\int \frac{dx}{e^x - 1}$

29. Find the volume of the solid generated by rotating the area bounded by the curve $y = \cos\frac{1}{2}x$ from $x = 0$ to $x = \pi$ about the $x-$ axis.

30. Evaluate: $\int_0^{\pi/3} x \sin 3x \, dx$

31. Prove that: $\int_{\pi}^{4\pi/3} \operatorname{cosec}\frac{1}{2}x \, dx = In 3$

32. Find the general solution of the d.e: $\frac{dy}{dx} + 2y = e^{-2x}\cos x$

33. Solve the differential equation: $x^2\frac{dy}{dx} - xy = 3y^2$ given $y = 1$ when $x = 1$.

34. Given that $y = \tan xy$, show that $\frac{dy}{dx} = \frac{y}{\cos^2 xy - x}$

35. Sketch the curve $y = \frac{3x + 3}{x(3 - x)}$, stating the intercepts and find the turning points.

TRIGONOMETRY

36. Given that $x=\sin(\theta+105^o)+\sin(\theta-15^o)+\sin(\theta+45^o)$ and $y=\cos(\theta+105^o)+\cos(\theta-15^o)+\cos(\theta+45^o)$, show that $\frac{x}{y}=\tan(\theta+45^o)$.

37. Prove in a triangle ABC given $\frac{a^2+b^2+c^2}{8R^2}=1+\cos A\cos B\cos C$ where R is the radius of the circumscribing circle.

38. Eliminate θ between the equations $x=a\cos^2\theta+b\sin^2\theta$ and $y=(a-b)\sin\theta\cos\theta$.

39. Solve the equation $3\sec^2\frac{x}{2}=\tan\frac{x}{2}+5$ for $0^0\le x\le 360^0$.

40. The roots of the equation $ax^2+bx+c=0$ are $\tan\alpha$ and $\tan\beta$. Express $\sec(\alpha+\beta)$ in terms of a, b, c.

41. Prove that: $\cos 3x=4\cos^3 x-3\cos x$, hence, solve the equation $1+\cos 3x=\cos x(1+\cos x)$ for $0^o\le x\le 360^o$.

42. If ABC is a triangle, prove that $\sin^2 A+\sin^2 B+\sin^2 C=2+2\cos A\cos B\cos C$.

43. Prove that $\sin\left(2\sin^{-1}x+\cos^{-1}x\right)=\sqrt{(1-x^2)}$.

44. Prove that: $\tan^{-1}\frac{1}{2}-\cos ec^{-1}\frac{\sqrt{5}}{2}=\cos^{-1}\frac{4}{5}$.

45. Solve: $\cos x-\sin x+\cos 3x-\sin 3x=0$ for $0^o\le x\le\pi$.

46. Express $10\sin x\cos x+12\cos 2x$ in the form $R\sin(2x+\alpha)$, hence or otherwise solve $10\sin x\cos x+12\cos 2x+7=0$ in the range $0^o\le x\le 360^o$.

47. Prove that: $\frac{\sin 8\theta\cos\theta-\sin 6\theta\cos 3\theta}{\cos 2\theta\cos\theta-\sin 3\theta\sin 4\theta}=\tan 2\theta$.

GEOMETRY

48. Prove that the equation of the tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point $(a\sec\theta,\ b\tan\theta)$ is $by+ax\sin\theta=(a^2+b^2)\tan\theta$. If the normal meets the $x-$ axis at A and the $y-$ axis at B, find the locus of the mid point of AB.

49. Find the equation of the tangent and the normal to the curve $xy=c^2$ at the point $P\left(ct,\ \frac{c}{t}\right)$. Given that the normal at P meets the curve again at Q, find the coordinates of Q. If the tangent at P meets the $y-$axis at R, find the equation of the locus of the midpoint M of PR.

50. Find the area of a quadrilateral $ABCD$, with coordinates $A(5,\ -2)$, $B(8,\ 2)$, $C(3,\ 7)$ and $D(3,\ \ 4)$.

51. Find the angle between the lines $3y=2x+5$ and $5y+2x=7$.

52. Given that $r=3\cos\theta$ is an equation of a circle, find the cartesian equation.

53. The normal to the parabola $y^2=4ax$ at the point $P(at^2,\ 2at)$ meets the axis of the parabola at G and GP is produced, beyond P to Q so that $\overline{GP}=\overline{PQ}$. Show that the equation of locus of Q is $y^2=16a(x+2a)$.

54. Find the equation of a circle whose centre lies on the line $y=3x-1$ and passes through the points $(1,\ 1)$ and $(2,\ -1)$.

55. The gradient of the side PQ of the rectangle $PQRS$ is $\frac{3}{4}$. The coordinates of the opposite corners Q, S are respectively $(6,\ 3)$ and $(-5,\ 1)$. Find the equation of PR.

56. The sides AB and BC of a parallelogram $ABCD$ have equations $y+3x=1$ and $y=5x-7$ respectively. If the coordinates of D are $(5,\ 10)$, find the coordinates of A, B, C.

VECTORS

57. With respect to the origin O, the points P, Q, R, S have position vectors given by $\overrightarrow{OP} = \mathbf{i} - \mathbf{k}$, $\overrightarrow{OQ} = -2\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OR} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\overrightarrow{OS} = 3\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$.

i) Find the equation of the plane containing P, Q and R

ii) The point N is the foot of the perpendicular from S to this plane. Find the position vector of N and show that the length of SN is 7.

58. The straight line l passes through the points with coordinates $(-5,\ 3,\ 6)$ and $(5,\ 8,\ 1)$. The plane p has equation $2x - y + 4z = 9$.

i) Find the coordinates of the point of intersection of l and p.

ii) Find the acute angle between l and p.

59a) Find the acute angle between the line whose vector equation is $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \boldsymbol{\mu}(2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and the plane $\mathbf{r}\,.\,(2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}) = 18$.

b) Find the position vector of the point where the line $\mathbf{r} = -\mathbf{i} - 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$ cuts the plane $\mathbf{r}\,.\,(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = -5$.

60. Two lines have vector equations $\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$, find the position vector of the point of intersection of the two lines and the Cartesian equation of the plane containing the two lines.

61i) Find the perpendicular distance of the point $(3,\ 0,\ 1)$ from the line $\dfrac{x-1}{3} = \dfrac{y+2}{4} = \dfrac{z}{12}$.

ii) Find the angle between the line $x - 1 = \dfrac{y-2}{0} = \dfrac{z-3}{3}$ and the plane $3x + 2y + z = 1$.

iii) Determine in dot product form the equation of a plane containing the point $(3,1,2)$ and the line $x - 1 = \dfrac{y-2}{0} = \dfrac{z-3}{3}$.

END